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*Original Citation:*

*Availability:*

This version is available <http://hdl.handle.net/2318/137578> since

*Published version:*

DOI:10.1387/theoria.4627

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# Logical Form and Truth-Conditions\*

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Received: 10.2.2012  
Final Version: 3.5.2012

BIBLID [0495-4548 (2013) 28: 78; pp. 439-457]  
DOI: 10.1387/theoria.4627

**ABSTRACT:** This paper outlines a truth-conditional view of logical form, that is, a view according to which logical form is essentially a matter of truth-conditions. Section 1 provides some preliminary clarifications. Section 2 shows that the main motivation for the view is the fact that fundamental logical relations such as entailment or contradiction can formally be explained only if truth-conditions are formally represented. Sections 3 and 4 articulate the view and dwell on its affinity with a conception of logical form that has been defended in the past. Sections 5-7 draw attention to its impact on three major issues that concern, respectively, the extension of the domain of formal explanation, the semantics of tensed discourse, and the analysis of quantification.

**Keywords:** Logical form; truth-conditions

**RESUMEN:** Este artículo esboza una concepción veritativo-condicional de la forma lógica, es decir, una concepción de acuerdo con la cual la forma lógica es esencialmente una cuestión de condiciones de verdad. La sección 1 proporciona algunas clarificaciones preliminares. La sección 2 muestra que la principal motivación para esta concepción es el hecho de que hay relaciones lógicas fundamentales, como la implicación o la contradicción, que sólo pueden explicarse formalmente si las condiciones de verdad se representan formalmente. Las secciones 3 y 4 articulan dicha concepción y profundizan en su afinidad con una concepción de la forma lógica que ha sido defendida en el pasado. Las secciones 5 a 7 destacan su impacto sobre tres asuntos principales que conciernen, respectivamente, a la extensión del dominio de las explicaciones formales, la semántica del discurso temporalizado, y el análisis de la cuantificación.

**Palabras clave:** forma lógica; condiciones de verdad

## 1. *Introduction*

Logical form has always been a primary concern of philosophers belonging to the analytic tradition. From the very beginning of the analytic tradition, the study of logical form has been privileged as a method of investigation, trusting to its capacity to reveal the structure of thought or the nature of reality. This paper deals with the question of what is logical form, which is directly relevant to any principled reflection on that method.

The view that will be outlined provides one answer to the question of what is logical form. One answer does not mean “the” answer. The thought that underlies

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\* Previous versions of this paper were given as talks in Turin and in Barcelona in the spring of 2010. I would like to thank the people who discussed with me on those occasions, in particular Diego Marconi, Genoveva Martí and Manolo Martínez for their helpful remarks. I also owe an improvement on the first section to the comments of an anonymous referee.



the paper is that there is no such thing as the correct view of logical form. The use of the term ‘logical form’ may be motivated by different theoretical purposes, and it should not be taken for granted that a unique notion of logical form can satisfy all those purposes. Different views of logical form might be equally acceptable for different reasons.

The paper will focus on one theoretical purpose, namely, the purpose of formal explanation. It is natural to expect that logical form plays a key role in formal explanation. A logical relation, such as entailment or contradiction, is formally explained if its obtaining or not in a given case is deduced from some formal principle that applies to that case in virtue of the logical form of the sentences involved. So it is natural to ask how is logical form to be understood in order to fit such a role. The next section suggests that, as a first step towards an answer, a contrast must be recognized between a widely accepted presumption about logical form and the purpose of formal explanation. The presumption, call it the *intrinsicity presumption*, is that logical form is an intrinsic property of sentences.

Two clarifications are in order. The first is that ‘intrinsic’ is intended to qualify a stable property of a sentence *s* that does not depend on the way *s* is used on a particular occasion and do not involve the semantic relations that tie *s* to other sentences used on that occasion or on other occasions. Suppose that the following sentences are uttered to say that Max is walking to an intended financial institution:

- (1) He is going to the bank
- (2) Max is going to the bank

An intrinsic property of (1) is the property of attributing some sort of movement to Max. Instead, a property of (1) that is not intrinsic is the property of being about the same person (2) is about<sup>1</sup>.

The second clarification is that ‘sentence’ is to be read as ‘disambiguated sentence’. Usually, a distinction is drawn between two stages of the understanding of a sentence *s*. One, disambiguation, concerns the assignment of a reading to *s*. The other, determination of content, presupposes disambiguation and concerns the assignment of a truth-condition to *s* depending on the context of utterance. Thus in the case of (1) the selection of a meaning for ‘bank’ pertains to the first stage, while the specification of a denotation for ‘he’ pertains to the second. According to the intrinsicity presumption, or at least to a shared version of it, ascription of logical form to sentences presupposes the first stage, although it is independent of the second<sup>2</sup>.

At least one well-understood notion of logical form - call it the *syntactic notion* - rests on the intrinsicity presumption. According to that notion, a logical form or

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<sup>1</sup> This may be regarded as part of the more general distinction drawn in Fine (2009, 22), between intrinsic and extrinsic semantic features of expressions.

<sup>2</sup> Sometimes disambiguation is called “pre-semantic” and determination of content is called “semantic”. But this terminology may be misleading, as what follows is neutral with respect to the question of whether determination of content is purely semantic or includes pragmatic factors.

LF is a syntactic representation associated with a sentence, which differs from the surface structure of the sentence and is the output of disambiguation. For example, the representation corresponding to (1) on the occasion considered includes a non-ambiguous lexical item that amounts to a reading of ‘bank’ as financial institution<sup>3</sup>.

If there is a contrast between the intrinsicity presumption and the purpose of formal explanation, as it will be suggested, the syntactic notion is not suitable for that purpose. The notion of logical form that will be considered, the *truth-conditional notion*, does not rest on the intrinsicity presumption. This is not to say that there is something wrong with the syntactic notion. Arguably, the syntactic notion is justified by different theoretical purposes, in that it can be employed in a systematic and empirically grounded theory of meaning. But it is important to recognize the difference between the two notions, because there are substantive issues that crucially depend on how logical form is understood. An elucidation of the implications of the truth-conditional view may help to get a better understanding of those issues.

## 2. *Relationality in formal explanation*

The contrast between the intrinsicity presumption and the purpose of formal explanation may be stated as follows. If the intrinsicity presumption holds, then every sentence has its own logical form. That is, for every sentence *s*, there is a formula - a sequence of symbols of some artificial language - that expresses the logical form of *s*. However, formal explanation requires that the formal representation of sentences is relational, in that the formula assigned to each sentence does not depend simply on the sentence itself, but also on the relation that the sentence bears to other sentences in virtue of the respective truth-conditions. Some examples will help illustrate.

*Case 1.* Let A be the following argument:

(3) This is a philosopher  
(3) This is a philosopher

Imagine that I utter A pointing my finger at Max as I say ‘this’ the first time and at José as I say ‘this’ the second time. There is a clear sense in which A is invalid so understood, namely, it is possible that its premise is true and its conclusion false. Assuming that entailment amounts to necessary truth-preservation, this means that its premise does not entail its conclusion. However, if there is a unique formula  $\alpha$  that expresses the logical form of (3), the only form that can be associated with A

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<sup>3</sup> Stanley (2000) spells out the syntactic notion. Borg (2007, 62-73) openly defends the intrinsicity presumption. Independently of the syntactic notion, any notion according to which logical form is individuated in terms of some intrinsic property of a sentence, say, the linguistic meaning of the expressions occurring in it, entails the intrinsicity presumption. Therefore, the following discussion relates to any such notion.

is the valid form  $\alpha; \alpha$ , where the semicolon replaces the horizontal line.

*Case 2.* Let B be the following argument:

- (4) This is different from this  
 (5) There are at least two things

Imagine that I utter B pointing my finger at Max as I say ‘this’ the first time and at José as I say ‘this’ the second time. B seems valid so understood, and it is plausible to expect that its validity can be derived from a structural analogy with ‘Max is different from José; therefore there at least two things’. But if a unique formula  $\alpha$  expresses the logical form of (4), there is no formal explanation of this fact. For the difference between the two occurrences of ‘this’ is not captured.

*Case 3.* Consider the following sentences:

- (6) I’m a philosopher  
 (7) I’m not a philosopher

Imagine that I utter (6) and you utter (7). Even though (7) is the negation of (6), in that it is obtained by adding ‘not’ to (6), there is a clear sense in which we are not contradicting each other: the things we say can both be true, or false. But if there is a unique formula  $\alpha$  that expresses the logical form of (6), there must be a unique formula  $\sim \alpha$  that expresses the logical form of (7). So the apparent absence of contradiction is not formally explained.

*Case 4.* Imagine that I utter (6) and you utter the following sentence pointing at me:

- (8) You are not a philosopher

Even though (8) is not the negation of (6), there is a clear sense in which we are contradicting each other: the things we say can’t both be true, or false. But if there is a unique formula  $\alpha$  for (6) and, similarly, there is a unique formula  $\beta$  for ‘You are a philosopher’, then the logical form of (8) is  $\sim \beta$ , where  $\beta$  differs from  $\alpha$ . So the apparent contradiction is not formally explained.

Let L be a first-order language whose vocabulary includes the connectives  $\sim, \supset, \wedge$ , the quantifiers  $\forall, \exists$ , a denumerable set of variables  $x, y, z, \dots$ , a denumerable set of constants  $a, b, c, \dots$ , and a denumerable set of predicates  $P, Q, R, \dots$ . The point about relationality that emerges from cases 1-4 turns out clear if we take L as our formal language. Consider case 1. If A is represented as  $Fa; Fb$ , its invalidity is formally explained. The fact, however, is that such representation makes explicit a semantic relation between the two utterances of (3). What justifies the assignment of  $a$  in the first case and  $b$  in the second is that ‘this’ refers to different things in the two cases, and consequently (3) gets different truth-conditions. This is not something that can be detected from (3) itself. No analysis of the intrinsic properties of

(3) can justify the conclusion that the constant to be assigned in the second case must differ from  $a$ .

Note that the point about relationality does not boil down to the old point that propositions are the real terms of logical relations because they are the real bearers of truth and falsity. The same problem may arise with propositions. Suppose that in case 1 two distinct propositions  $p_1$  and  $p_2$  are assigned to the two occurrences of (3). If logical forms are understood as properties of propositions individuated in terms of some general feature that  $p_1$  and  $p_2$  share, such as being about an object that satisfies a condition, and this feature is expressed by a formula  $\alpha$ , then  $\alpha$  is to be assigned to both occurrences of the sentence. For example,  $Fa; Fa$  or  $Fb; Fb$  could be adopted as formal representations of A. In what follows, talk about propositions will be avoided, as the point about relationality can be phrased simply in terms of sentences and truth-conditions.

Note also that the explanatory shortcoming illustrated by cases 1-4 arises not only if it is supposed that the logical form of a sentence is expressed by a single formula, but also if it is supposed, for some set of formulas, that the logical form of the sentence is expressed by any member of the set, or by the set itself. As long as truth-conditions are not taken into account, there is no intelligible way to justify an appropriate choice of members of the set. Take case 1 again. Even if the supposition is that the logical form of (3) is expressed by a set of formulas  $Fa, Fb, \dots$ , there is no way to justify an assignment of different constants  $a$  and  $b$  in the two cases. No distinction can be drawn between  $Fa; Fb$  and  $Fa; Fa$  unless some semantic relation between the two utterances of (3) is taken into account.

In substance, any notion of logical form based on the intrinsicity presumption will yield the same result: as long as truth-conditions aren't formally represented, logical relations aren't formally explained. Certainly, one might simply stick to the intrinsicity presumption and accept the result. After all, a notion of logical form based on the intrinsicity presumption may be motivated on the basis of theoretical purposes other than formal explanation. But that option will not be considered here. In what follows it will be suggested that a different notion of logical form, the truth-conditional notion, can fit the purpose of formal explanation.

### 3. *Formalization and interpretation*

The line of thought that substantiates the truth-conditional notion rests on three assumptions. The first expresses a constraint on formalization that is commonly taken for granted in logic textbooks. Since a representation of a set of sentences in a formal language is intended to provide an account of the possible truth-values of the sentences in the set, it is usually expected that the relations of identity and difference between their truth-conditions are to be made explicit. Consider a representation of the following sentences in L:

(9) Snow is white

(10) Grass is green

As undergraduate logic students know, different formulas must be assigned to (9) and (10), say  $Fa$  and  $Ga$ . For (9) and (10) have different truth-conditions, so one of them might be true and the other false. By contrast, consider a representation in  $L$  of (9) and

(11) Snow is indeed white

In this case the same formula, say  $Fa$ , can be used for both sentences. For no possible arrangement of truth-values will fail to be represented. Let us stipulate that, for two  $n$ -tuples  $\bar{x}$  and  $\bar{y}$ ,  $\bar{y}$  mirrors  $\bar{x}$  if and only if for every  $i$  and  $k$  between 1 and  $n$ ,  $x_i = x_k$  if and only if  $y_i = y_k$ . The constraint may then be stated in general terms:

(C) Given an  $n$ -tuple of sentences  $\bar{s}$  with truth-conditions  $\bar{t}$ , an  $n$ -tuple of formulas  $\bar{\alpha}$  adequately formalizes  $\bar{s}$  only if it mirrors  $\bar{t}$ .

The expression ‘an  $n$ -tuple of sentences  $\bar{s}$  with truth-conditions  $\bar{t}$ ’ will be read as entailing that  $\bar{t}$  can be ascribed to  $\bar{s}$  depending on the states of affairs that the sentences in  $\bar{s}$  describe as obtaining. This is just a rough characterization and certainly it does not settle every issue concerning sameness of truth-conditions. But at least it provides a rationale for accepting clear cases of identity or difference between truth-conditions. For example, (9) and (11) have the same truth-condition, for they describe the same object, snow, as having the same property, that of being white. By contrast, (9) and (10) have different truth-conditions, because (10) describes a different object, grass, as having a different property, that of being green. Given (C), the first assumption may be phrased as follows:

(A1) Formulas mirror truth-conditions.

That is, for every  $\bar{s}$  with truth-conditions  $\bar{t}$  and every  $\bar{\alpha}$  that adequately formalizes  $\bar{s}$ ,  $\bar{\alpha}$  mirrors  $\bar{t}$ .

The second assumption boils down to a generally accepted fact, namely, that there is no correspondence between sentences and truth-conditions: the same sentence may have different truth-conditions, and different sentences may have the same truth-conditions. Indexicals provide paradigmatic examples in this respect. (6) uttered by me and (6) uttered by you have different truth-conditions, while (7) uttered by me and (8) uttered by you have the same truth-condition. Again, the rationale is that in the first case the same property is attributed to different objects, while in the second the same property, or its absence, is attributed to the same object. More generally, it is not the case that, for every  $\bar{s}$  with truth-conditions  $\bar{t}$ ,  $\bar{t}$  mirrors  $\bar{s}$ :

(A2) Truth-conditions do not mirror sentences.

The third assumption rests on a relatively uncontroversial claim, namely, that the logical form of a set of sentences is expressed by an adequate formalization of the set. The assumption may be stated as follows:

(A3) Logical forms mirror formulas.

That is, if  $\bar{s}$  is adequately formalized by  $\bar{\alpha}$ , then the logical form of each sentence in  $\bar{s}$  is expressed by the corresponding formula in  $\bar{\alpha}$ . (A3) amounts to a strict reading of the uncontroversial claim, although it is not the only possible reading.

(A1) and (A2) entail that it is not the case that, for every  $\bar{s}$  and every  $\bar{\alpha}$  that adequately formalizes  $\bar{s}$ ,  $\bar{\alpha}$  mirrors  $\bar{s}$ . That is,

(12) Formulas do not mirror sentences.

Mirroring is an equivalence relation, so if  $\bar{\alpha}$  mirrors  $\bar{t}$  but  $\bar{t}$  does not mirror  $\bar{s}$ , then  $\bar{\alpha}$  does not mirror  $\bar{s}$ . From (A3) and (12) it follows that it is not the case that for every  $\bar{s}$ , every  $n$ -tuple of logical forms expressed by an adequate formalization of  $\bar{s}$  mirrors  $\bar{s}$ . That is,

(13) Logical forms do not mirror sentences.

(13) suggests that there is no such thing as “the” logical form of a sentence. Sometimes a negative claim of this kind is held on the basis of entirely different motivations, which involve skepticism about uniqueness of formal representation. It is argued that the same sentence  $s$  can equally be associated with different formulas that belong to different languages. For example, (9) can be translated in  $L$  as  $Fa$ , while in a propositional language it gets a sentential variable. Similarly, it is argued that  $s$  can equally be associated with different formulas of the same language. Consider

(14) José is older than Max

In  $L$  there are at least two ways to represent (14), that is,  $Fa$  and  $Rab$ .  $F$  stands for ‘older than Max’, while  $R$  stands for ‘older than’<sup>4</sup>.

It is an open issue whether skeptical arguments of this ilk are grounded. In both cases, the skeptic maintains that there are at least two equal representations of  $s$ . Yet much depends on what ‘equal’ is supposed to mean. It might be contended that a translation of (9) in  $L$  and a translation of (9) in a propositional language are not equal: the first is a finer representation, in that it provides an analysis of

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<sup>4</sup> Considerations along these lines are offered in Brun (2008, 6).



the structure of the content of (9). Similarly, it might be contended that  $Fa$  and  $Rab$  are not equal translations of (14):  $Rab$  is finer, in that it makes explicit the relational character of the property ascribed to José. Anyway, this issue will not be addressed here. The cogency of the reasoning that leads to (13) can be recognized independently of any commitment to skepticism about uniqueness of formal representation. One might coherently accept (13) and hold that there is such a thing as the finest representation of  $s$ .

The only sense to be considered in which there is no such thing as “the” logical form of a sentence is the following. Assuming that an *interpretation* of  $s$  fixes a truth-condition for  $s$ , in that it includes both a disambiguation of  $s$  and a determination of its content,  $s$  has logical form only relative to this or that interpretation. Similarly,  $s$  and  $s'$  can be said to have the same logical form, or different logical forms, only relative to this or that interpretation. Sentences are formalized in virtue of the relations of identity and difference between their truth-conditions, so the question whether two sentences have the same logical form depends on such relations<sup>5</sup>.

The view of logical form that emerges from the line of thought set out may be called the *truth-conditional view*, in that it implies that the logical form of a sentence  $s$  in an interpretation  $i$  is determined by the truth-condition that  $s$  has in  $i$ . Let us assume that, for any  $n$ -tuple of sentences  $\bar{s}$ , an *interpretation* of  $\bar{s}$  is an  $n$ -tuple  $\bar{i}$  such that each term in  $\bar{i}$  is an interpretation of the corresponding term in  $\bar{s}$ . A general definition may be given as follows:

**Definition 1.**  $\bar{s}$  has logical form  $\bar{\alpha}$  in  $\bar{i}$  iff  $\bar{s}$  is adequately formalized by  $\bar{\alpha}$  in  $\bar{i}$ .

If  $\bar{s}$  has exactly one term, we get that  $s$  has logical form  $\alpha$  in  $i$  if and only if  $s$  is adequately formalized by  $\alpha$  in  $i$ .

Cases 1-4 may be handled in accordance with definition 1. On the assumption that every truth of first-order logic is a formal principle, in each case the obtaining or not of a relation of entailment or contradiction is deducible from some formal principle that applies to that case. Consider case 1. In the intended interpretation of A, ‘this’ refers to Max in the premise and to José in the conclusion. Therefore, an adequate formalization of A in that interpretation is  $Fa; Fb$ . Since  $Fa; Fb$  is not a valid form, the apparent lack of entailment is formally explained. That is, A is formally invalid in the intended interpretation. Now consider case 2. The intended interpretation of B is one according to which B is adequately formalized as  $a \neq b; \exists x \exists y (x \neq y)$ . This is a valid form, that is, the same exemplified by ‘Max is

<sup>5</sup> This has nothing to do with relationism about meaning, the view defended in Fine (2009). According to Fine (2009) “the fact that two utterances say the same thing is not entirely a matter of their intrinsic semantic features; it may also turn on semantic relationships among the utterances or their parts which are not reducible to those features. We must, therefore, recognize that there may be *irreducible* semantic relationships, ones not reducible to the intrinsic semantic features of the expressions between which they hold” (p. 3). The individuation of the logical form of two sentences may be relational even if their meaning is fixed in some non-relational way.

different from José; therefore, there are at least two things'. So the apparent entailment is formally explained. Cases 3 and 4 are analogous. In case 3, (6) and (7) are adequately formalized as  $Fa, \sim Fb$  in the intended interpretation. So there is no contradiction, just as it should be. In case 4, (6) and (8) are adequately formalized as  $Fa, \sim Fa$  in the intended interpretation, so they do contradict each other.

From definition 1 it turns out that the primary sense in which sentences have logical form is that in which they have logical form relative to interpretations. But the primary sense need not be the only sense. Talk of logical form relative to interpretations does not rule out talk of logical form *simpliciter*. The reason we may have to talk of logical form *simpliciter* is quite clear. Some semantic properties of sentences are neutral with respect to the specific contents conveyed by them. For example, the contents conveyed by (3) in different contexts have something in common. So it is plausible that a unique form can be ascribed to (3), even though two different utterances of (3) can be translated as  $Fa$  and  $Fb$ . Talk of logical form in this sense is acceptable, if logical form *simpliciter* is understood in terms of quantification over logical forms relative to interpretations. That is,

**Definition 2.**  $\bar{s}$  has a given logical form iff  $\bar{s}$  has that logical form in all interpretations.

Definitions 1 and 2 may be regarded as different ways of spelling out the claim that the logical form of a set of sentences is expressed by an adequate formalization of the set. Definition 1, in accordance with the reading of the claim that is called 'strict' in §3, entails that the logical form of  $\bar{s}$  relative to  $\bar{i}$  is given by the very formulas assigned to  $\bar{s}$  in  $\bar{i}$ . For example, if a pair of sentences is adequately formalized as  $Fa, Fa$  in an interpretation, the logical form of the pair in the interpretation is just  $Fa, Fa$ . Definition 2, instead, entails that the logical form *simpliciter* of  $\bar{s}$  is given by the kind of formulas assigned to  $\bar{s}$  in its interpretations, where the meaning of 'kind' can be specified in the metalanguage that describes the syntax of the language in which logical forms in the first sense are expressed. For example, if a pair of sentences is adequately formalized as  $Fa, Fa$  (or  $Fb, Fb$ , and so on) in all interpretations,  $Pt, Pt$  can be ascribed to the pair as logical form *simpliciter*, where  $P$  is a variable for unary predicates of L and  $t$  is a variable for closed terms of L. The same goes for  $\alpha; \alpha$ , if  $\alpha$  stands for any formula of L.

Note that definition 2 introduces a stable property of sentences of the sort envisaged by those who rely on the intrinsicity presumption. If  $Pt$  can be ascribed to (3) *simpliciter*, then there is a unique logical form for (3). Yet this is not quite the same thing as to say that the definition entails the intrinsicity presumption. First of all, any ascription of logical form *simpliciter* to  $s$  depends on prior ascriptions of logical form relative to interpretations of  $s$ . So its ultimate ground is the understanding of the truth-conditions of  $s$  in those interpretations. Secondly, definition 2 does not entail that every sentence has logical form *simpliciter*, or even that some sentence has logical form *simpliciter*. It is consistent with definition 2 to maintain that some (or even all) sentences do not have stable logical forms, as their logical form vary with interpretation.

#### 4. *The idea that distinct symbols name distinct objects*

The truth-conditional view is reminiscent of a conception of logical form once defended by Russell and Wittgenstein. According to that conception, the logical form of a sentence can at least in principle be displayed in a logically perfect language that represents states of affairs by means of symbols that denote their constituents. So the idea that distinct symbols name distinct objects is at the core of that conception. The truth-conditional view rests on the same idea, as it turns out clear from the way the constants *a* and *b* are employed in cases 1-4<sup>6</sup>.

Almost nobody now regards the notion of a logically perfect language as a guide to the understanding of logical form. Several troubles have been raised in connection with that notion, and it is a shared feeling that at least some of them are too serious to be neglected. One well-known problem is the following. A logically perfect language must include a name for every object. But none of the formal languages we normally employ has this feature. Presumably, the number of objects exceeds that of non-logical symbols of any such language. Another problem concerns the nature of the objects named by the symbols of the logically perfect language. Those objects are taken to be simple. Therefore, it seems that no ordinary object, such as a person, can be the denotation of a symbol. This means that the relation between ordinary language and the logically perfect language is so complex that logical form becomes ineffable<sup>7</sup>.

However, it would be wrong to think that any problem that affects the conception of logical form based on the notion of a logically perfect language poses a threat to the truth-conditional view. For the truth-conditional view does not involve such a notion. Instead of appealing to an ideal language of which we know nothing, the view entails that logical forms can be expressed in a familiar language of which we know everything, namely *L*. Certainly, *L* does not include a constant for every object, at least on the assumption that there is a non-denumerable infinity of objects. But this is not a problem, as no fixed relation is taken to obtain between constants and objects. Logical forms are ascribed to sentences relative to interpretations, so it is only relative to interpretations that constants denote objects. Consider case 1. In the intended interpretation, the objects in need of a name are Max and José, so the constants *a* and *b* suffice for the purpose of formal representation. It doesn't really matter how many objects are left without a name in that interpretation. Similar considerations hold for the second problem. Nothing prevents *a* and *b* from denoting ordinary objects such as persons in one interpretation, and simpler objects of a different kind in another interpretation.

A more pertinent objection that might be raised against the truth-conditional view concerns the very idea that distinct symbols name distinct objects. That idea, it might be contended, widens the domain of semantic competence in unaccept-

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<sup>6</sup> The old conception of logical form emerges clearly in Russell (1998, 58) and in Wittgenstein (1992, 139).

<sup>7</sup> It is not entirely clear whether Russell and Wittgenstein have this second problem, as it is not entirely clear whether they understand simplicity as an absolute property.

able way, for it entails that knowledge of reference is part of semantic competence. Consider the following sentences:

- (15) Hesperus is a star  
 (16) Phosphorus is a star

Since ‘Hesperus’ and ‘Phosphorus’ refer to the same planet, (15) and (16) must have the same logical form in any interpretation, say *Fa*. But one needs substantive empirical information to know that ‘Hesperus’ and ‘Phosphorus’ refer to the same planet. So it turns out that one needs substantive empirical information to grasp the logical form of (15) and (16).

This objection, however, is far from decisive. First of all, it is not patently false that knowledge of reference is part of semantic competence. At least *prima facie*, it is tenable that semantic competence includes knowledge of the fact that ‘Hesperus’ and ‘Phosphorus’ refer to the same planet, if it is maintained that speakers can correctly use (15) or (16) without being fully competent. Secondly, and more importantly, even granting that knowledge of reference is not part of semantic competence, as the objection requires, it is questionable that the truth-conditional view entails the opposite conclusion. In order to get that conclusion, it should be assumed in addition that knowledge of logical form is part of semantic competence. But the additional assumption may be rejected. On the truth-conditional view, it may be claimed that semantic competence does not include knowledge of the fact that (15) and (16) have logical form *Fa*, just because the latter involves substantive empirical information that does not pertain to semantic competence.

Two remarks may be added to complete the reply to the objection. The first is that the truth-conditional view is not intended to provide an explanation of how speakers get to know truth-conditions. The view by no means entails that one has to “go through” the logical form of *s* in order to grasp the truth-condition of *s*. What is suggested is rather the contrary. The ascription of logical form to *s* in a given interpretation requires prior understanding of the truth-condition of *s* in that interpretation. One is in a position to adequately formalize (15) and (16) only when one knows, based on substantive empirical information, that ‘Hesperus’ and ‘Phosphorus’ refer to the same planet<sup>8</sup>.

The second remark is that, on the truth-conditional view, there is no interesting sense in which logical form is “transparent”. The logical form of *s* is the structure of the thing said by uttering *s*, hence it is something that may not be detectable from the surface grammar of *s*. This is not just to say there may be a systematic divergence between the surface grammar of *s* and its logical form that is knowable *a priori* as a result of semantic competence. The understanding of the logical form of *s* may involve empirical information that is not so knowable. Therefore, using *s* correctly by no means entails being in a position to know the logical form of *s*.

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<sup>8</sup> This is not to deny that some notion of logical form, such as the syntactic notion, can be invoked as part of an explanation of how a speaker is able to grasp the truth-condition of a sentence.

### 5. *The domain of formal explanation*

This section and the remaining two deal with three significant implications of the truth-conditional view. In each of the three cases the view entails that, contrary to what is often assumed, considerations about syntactic structure and linguistic meaning do not justify claims about logical form. The first implication concerns the extension of the domain of formal explanation, and may be resumed as follows: the relations that can formally be explained are not exactly those one might expect. There are cases in which it is commonly believed that formal principles do not apply but in reality they do apply, and cases in which it is commonly believed that formal principles apply but in reality they do not apply.

A case of the first kind is provided by the example of Hesperus and Phosphorous considered in §4. Let C be the following argument:

- |                            |  |
|----------------------------|--|
| (15) Hesperus is a star    |  |
| (16) Phosphorous is a star |  |

C is usually regarded as formally invalid, although the inference from (15) to (16) is necessarily truth-preserving. For it is assumed that C does not instantiate a valid form. However, this assumption fails on the truth-conditional view. In any interpretation, C can be represented as  $Fa; Fa$ , so it is formally valid. Moreover, if formal validity *simpliciter* is defined as formal validity in all interpretations, C is formally valid *simpliciter*.

At least two predictable ways to resist this conclusion are easily countered. In the first place, it is pointless to insist that C is formally invalid by appealing to some sort of transparency that is assumed to characterize logical necessity, as opposed to metaphysical necessity. As it turns out from §4, the truth-conditional notion of logical form does not justify such transparency assumption. Thus it is pointless to argue, say, that C is formally invalid because the connection between (15) and (16) is not detectable from the meaning of some “logical” words occurring in them.

In the second place, it cannot be contended that the truth-conditional view entails that formal validity reduces to necessary truth-preservation and so blurs the distinction between logical necessity and metaphysical necessity. For it is not essential to the view to assume that any two necessary sentences have the same truth-condition. It is consistent with the characterization of truth-conditions given in §3 to claim that two necessary sentences, such as ‘2 is even’ and ‘3 is odd’, may have different truth-conditions, hence that there may be no formal explanation of the validity of the inference from the former to the latter. In substance, the truth-conditional view entails at most that some necessarily truth-preserving arguments are formally tractable, which is something that anyone should accept<sup>9</sup>.

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<sup>9</sup> An alternative option would be to retain the assumption that any two necessary sentences have the same truth-condition and replace (C) with a weaker condition that makes room for the possibility that distinct but equivalent formulas are assigned to such sentences.

A case of the second kind is provided by the argument A considered in §2. Many would say that A is valid, in that it instantiates a valid form. Or at least, many would accept the following conditional:

(17) If  $\alpha; \alpha$  is a valid form then A is valid.

However, from §3 it turns out that (17) may be denied. Let us suppose, as standard logic demands, that  $\alpha; \alpha$  is a valid form. On the assumption that validity and logical form are properties that belong to arguments relative to interpretations, this is to say that no argument has logical form  $\alpha; \alpha$  in an interpretation but is invalid in that interpretation. Therefore, the supposition that  $\alpha; \alpha$  is a valid form is consistent with the claim that A is invalid in an interpretation in which ‘this’ refers to different persons in the premise and in the conclusion. For in that interpretation A is correctly formalized as  $Fa; Fb$ , so it does not have logical form  $\alpha; \alpha$ . Since validity *simpliciter* may be understood as validity in all interpretations, the supposition that  $\alpha; \alpha$  is a valid form is equally consistent with the claim that A is invalid *simpliciter*.

One interesting corollary of this result concerns the question of how arguments affected by context-sensitivity are to be evaluated. A rather influential account of validity is due to Kaplan. Assuming that the contribution of a context to the truth-condition of a sentence can be described in terms of an index - a sequence of coordinates relative to which the sentence gets a content - Kaplan suggests that validity can be defined as truth-preservation for all indices. That is, an argument is valid if and only if, for every index, if its premises are true relative to that index, its conclusion must be true relative to that index. So A turns out valid on Kaplan’s account. Assuming that indices include a suitable coordinate for ‘this’, it is impossible that the same index makes (3) true and false at the same time<sup>10</sup>.

There are cases, however, that clash with Kaplan’s account. Case 1 is one of them, as it seems to be a case in which truth is not preserved. In general, a definition of validity that rules out index-shifts fails to account for cases in which the apparent validity or invalidity of an argument involves some index-shift in the intended interpretation. Nonetheless, Kaplan defends his account. If we allow that A is invalid due to the possibility of index-shifts, he argues, then we get the undesirable result that the forms they instantiate are invalid:

Thus even the most trivial of inferences, P therefore P, may appear invalid<sup>11</sup>.

<sup>10</sup> Kaplan’s account is the account proposed in Kaplan (1989b). Arguments similar to A may be phrased with “pure” indexicals, such as ‘now’.

<sup>11</sup> Kaplan (1989a, 584-585). Kaplan’s distinction between “occurrence” and “utterance” is not relevant here. One question is whether validity should be defined in terms of occurrence or in terms of utterance, another question is whether the definition should leave room for index-shifts. The two questions are independent in that the second remains open even if it is granted that validity is to be defined in terms of occurrence. No reference to the utterance of the argument is needed to take into account the possibility that (3) takes different indices.

An alternative to Kaplan's account has been proposed by Yagisawa, driven by the same problem that Kaplan's argument is intended to dismiss. Yagisawa claims that A is invalid, in that there are cases, such as case 1, in which the inference in A does not preserve truth. The consequence he draws is that  $\alpha; \alpha$  is not a valid form. Kaplan and Yagisawa seem to agree on the following dilemma: either one claims that A is valid and maintains that  $\alpha; \alpha$  is a valid form, or one claims that A is invalid and denies that  $\alpha; \alpha$  is a valid form. This dilemma, however, rests on (17), so it can be rejected if (17) is rejected. Without (17), the hypothesis that A is invalid cannot be ruled out by contraposition, so Kaplan's argument collapses. Similarly, Yagisawa's departure from standard logic becomes unnecessary, for the invalidity of the form  $\alpha; \alpha$  is not a consequence of that hypothesis<sup>12</sup>.

## 6. *Tensed sentences*

The second implication concerns the long-standing issue of what is the logical form of tensed sentences. Prior built tense logic on the assumption that tenses are to be treated as operators. Consider the following sentences:

- (18) José is a bachelor
- (19) José was a bachelor
- (20) José will be a bachelor

According to Prior's analysis, (18) is a simple sentence that can be evaluated as true or false at any time. That is, (18) is true at  $t$  if and only if José is a bachelor at  $t$ . (19), instead, is a more complex sentence that amounts to a combination of 'It was the case that' and (18). (20) is similar to (19), as it amounts to a combination of 'It will be the case that' and (18). Thus, (19) is true at  $t$  if and only if (18) is true at  $t'$  for some  $t'$  earlier than  $t$ , while (20) is true at  $t$  if and only if (18) is true at  $t'$  for some  $t'$  later than  $t$ <sup>13</sup>.

A classical alternative to Prior's analysis is the "extensional" analysis, that is, the analysis in terms of quantification over times. On that analysis, the content of (18) is appropriately stated as 'For some  $t$ ,  $t$  is the present time and José is bachelor at  $t$ '. The case of (19) and (20) is similar. (19) is analysed as 'For some  $t$ ,  $t$  is earlier than the present time and José is bachelor at  $t$ ', while (20) is analysed as 'For some  $t$ ,  $t$  is later than the present time and José is bachelor at  $t$ '. The extensional analysis, originally introduced for purely philosophical reasons, has gained acceptance in formal semantics in the course of its development as an autonomous discipline within linguistics. In the last few years, its increasing popularity has fomented the

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<sup>12</sup> Yagisawa's alternative is presented in Yagisawa (1993). Iacona (2010) outlines and defends an account of the relation between validity and context-sensitivity that is consistent with the rejection of (17).

<sup>13</sup> Prior's analysis goes back to Prior (1957).

debate between the friends of operators and their enemies<sup>14</sup>.

To get a sense of the relevance of the issue of logical form to this debate, it will suffice to focus on a specific argument that has been invoked in support of Prior's analysis. On the extensional analysis, the argument goes, the real content of (18) involves reference to times. But this makes the structure of (18) more complex than it appears. According to a version of the argument due to Kamp, the problem lies in the fact that no reference to abstract entities such as times seems involved in (18). According to another version due to Blackburn and Recanati, the problem lies in the departure from the "internal" perspective on time, that is, the perspective we have as speakers situated inside the temporal flow:

The present tense is not a tense like the past or the future. It is more primitive and, in a sense, temporally neutral. Someone can think 'It is hot in here' even if she has no notion of time whatsoever, hence no mastery of the past and the future<sup>15</sup>.

However, if logical form is a matter truth-conditions, no consideration concerning the apparent structure of (18) or the cognitive aspects pertaining to its use can rule out the hypothesis that the logical form of (18) is expressed by means of a quantification over times. The truth-conditions of tensed sentences may diverge from their surface grammar, and certainly transcend the "internal" perspective on time.

Of course, this is not to say that the arguments that have been provided in favour of the extensional analysis are all good. As a matter of fact, some of them are undermined by the truth-conditional view for similar reasons. Yet it is at least consistent with the view to suppose that the logical form of tensed sentences may be expressed in L. Thus, (18)-(20) may be formalized as follows:

(21)  $\exists x(x = a \wedge Rbx)$

(22)  $\exists x(x < a \wedge Rbx)$

(23)  $\exists x(a < x \wedge Rbx)$

Here  $<$  stands for 'earlier than',  $R$  stands for 'is bachelor at',  $a$  is a name for the present time, and  $b$  denotes José.

## 7. Quantification

The third implication concerns the analysis of quantification. One question that has been addressed within the framework of general quantification theory is whether first-order logic has the resources to express the logical form of quantified sentences.

<sup>14</sup> Evans (1985) is a *locus classicus* of the early philosophical resistance to tense logic. King (2003) is a recent linguistically oriented defence of the extensional analysis.

<sup>15</sup> Recanati (2007, 70). Similar considerations appear in Kamp (1971, 231), and in Blackburn (1994, 83).



Consider the following sentences:

- (24) All philosophers are rich  
 (25) Some philosophers are rich

(24) and (25) are quantified sentences, in that they contain the quantifier expressions ‘all’ and ‘some’. There is a standard way to formalize (24) and (25) in L:

- (26)  $\forall x(Px \supset Qx)$   
 (27)  $\exists x(Px \wedge Qx)$

However, not all quantified sentences are like (24) and (25). Consider the following, which differs from (24) and (25) in that it contains the quantifier expression ‘more than half of’:

- (28) More than half of philosophers are rich

Although (28) is semantically similar to (24) and (25), in that it is formed by expressions of the same semantic categories combined in the same way, there is no formula of L that translates (28) in the same sense in which (26) and (27) translate (24) and (25). Assuming that the meaning of a quantifier expression is a *quantifier*, that is, a function  $Q$  that assigns to each context  $c$  a binary relation  $Q_c$  between subsets of the intended domain  $D_c$ , the sense in question may be stated in terms of the following property of quantifiers:

**Definition 3.**  $Q$  is first-order definable iff there is a sentence  $\alpha$  of L containing two unary predicates such that, for every  $c$  and  $A, B \subseteq D_c$ ,  $Q_c(A, B)$  iff  $\alpha$  is true in a structure with domain  $D_c$  where the predicates in  $\alpha$  denote  $A$  and  $B$ .

Say that a quantifier expression is first-order definable if the quantifier it signifies is first-order definable. While ‘all’ and ‘some’ are first-order definable, ‘more than half of’ is not. The same goes for vague quantifier expressions such as ‘most’, ‘many’ and ‘few’. As in the case of (28), there is no translation in L of the following sentences:

- (29) Most philosophers are rich  
 (30) Many philosophers are rich  
 (31) Few philosophers are rich<sup>16</sup>

Many are inclined to think that this fact constitutes a serious limitation of the

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<sup>16</sup> First-order definability is as in Peters and Westerståhl (2006, 451), except for the reference to contexts. A proof of the first-order undefinability of ‘more than half of’ is provided in Barwise and Cooper (1981, 213-214). Peters and Westerståhl (2006) spells out a proof method that may be extended to other quantifier expressions such as ‘most’, ‘many’ and ‘few’, at least on a plausible reading of them, see pp. 466-468.

expressive power of first-order logic. If it is taken for granted that formalization is a matter of translation, understood as meaning preservation, then it is natural to believe that there is no way to formalize (28) in  $L$ . More generally, one may be tempted to think that a quantified sentence can be formalized in  $L$  only if the quantifier expressions it contains are first-order definable. Lepore and Ludwig regard sentences such as (29)–(31) as emblematic of the difficulty philosophers have been led into by thinking of paraphrase into a first-order language as the proper approach to exhibiting logical form. They argue as follows. Since (29)–(31) are semantically similar to (24) and (25), they must have similar logical forms. But there is no sentence of  $L$  similar to (26) or (27) that translates (29)–(31). Therefore, the logical form of (29)–(31) cannot be expressed in  $L$ <sup>17</sup>.

This line of argument, however, rests on the intrinsicity presumption, in that it assumes that logical form is a matter of meaning. On that assumption, semantic similarity amounts to similarity in logical form, and the fact that no sentence of  $L$  similar to (26) or (27) translates (28)–(31) may be invoked in support of the conclusion that no sentence of  $L$  can express the logical form of (28)–(31). But the truth-conditional view rejects the intrinsicity presumption. An alternative hypothesis that is consistent with the view is that the meaning of a quantified sentence is a function from contexts to truth-conditions, hence logical form can be ascribed to quantified sentences only relative to contexts. According to this hypothesis, it doesn't really matter whether there are sentences of  $L$  that have the same meaning of (28)–(31). Sameness of meaning is not the relevant issue, even in the case of (24) and (25). For formalization is not intended as translation, but as representation of truth-conditions.

In other words, it is debatable that first-order definability is the property to be considered in order to decide whether first-order logic has the expressive resources to deal with quantification in natural language. A different property may be defined as follows:

**Definition 4.**  *$Q$  is first-order expressible iff for every  $c$  and  $A, B \subseteq D_c$ , there is a sentence  $\alpha$  of  $L$  containing two unary predicates such that  $Q_c(A, B)$  iff  $\alpha$  is true in a structure with domain  $D_c$  where the two predicates denote  $A$  and  $B$ .*

The difference between definition 3 and definition 4 is that the former requires that there is an appropriate sentence of  $L$  that is the same for every context, while the latter requires that for every context there is an appropriate sentence of  $L$ . First-order definability entails first-order expressibility, but not the other way round. If  $Q$  is such that there is a sentence  $\alpha$  that expresses the claim that  $Q_c(A, B)$  for every  $c$ , then for every  $c$  there is a sentence that expresses the claim that  $Q_c(A, B)$ , namely,  $\alpha$  itself. Yet it may be the case that for different contexts  $c$  and  $c'$  there are distinct sentences  $\alpha$  and  $\alpha'$  that adequately represent the claims that  $Q_c(A, B)$  and  $Q_{c'}(A, B)$ .

Since a quantifier expression can be first-order expressible without being first-order definable, the fact that 'more than half of' is not first-order definable does

<sup>17</sup> Lepore and Ludwig (2002, 70). See also Barwise and Cooper (1981, 159).

not entail that it is not first-order expressible. The same goes for ‘most’, ‘many’ and ‘few’. In the case of ‘more than half’, it is easy to see that first-order expressibility holds if it is assumed, in accordance with ordinary use, that domains are finite. Suppose that (28) is uttered in a context  $c$  in which there are exactly three philosophers. In this case the assertion made is that at least two of them are rich. Suppose instead that (28) is uttered in a context  $c'$  in which there are exactly four philosophers. In this case the assertion made is that at least three of them are rich. Therefore, different sentences of  $L$  adequately represent the truth-conditions of (28) in  $c$  and in  $c'$ , that is:

$$(32) \exists x \exists y (x \neq y \wedge Px \wedge Qx \wedge Py \wedge Qy)$$

$$(33) \exists x \exists y \exists z (x \neq y \wedge x \neq z \wedge y \neq z \wedge Px \wedge Qx \wedge Py \wedge Qu \wedge Pz \wedge Qz)$$

More generally, (28) has different logical forms in different contexts, each of which is expressible in  $L$ . The case of ‘most’, ‘many’ and ‘few’ may be described in similar way, if it is assumed that the vagueness of a quantifier expression consists in its capacity in principle to be made precise in more than one way. For contexts may be taken to involve precisifications of quantifiers in addition to intended domains, so that in each context, a sentence such as (29)-(31) has a definite truth-condition representable in  $L$ <sup>18</sup>.

## REFERENCES

- Barwise, J., and R. Cooper. 1981. Generalized Quantifiers and Natural Language. *Linguistics and Philosophy* 4: 159–219.
- Blackburn, P. 1994. Tense, Temporal Reference and Tense Logic. *Journal of Semantics* 11: 83–101.
- Borg, E. 2007. *Minimal semantics*. Blackwell.
- Brun, G. 2008. Formalization and the Objects of Logic. *Erkenntnis* 69: 1–30.
- Evans, G. 1985. Does Tense Logic Rest upon a Mistake? In *Collected Papers*, 343–363. Clarendon Press.
- Fine, K. 2009. *Semantic relationism*. Blackwell.
- Iacona, A. Forthcoming. Quantification and Logical Form. In A. Torza (ed.), *Quantifiers, Quantifiers, and Quantifiers*, Springer.
- Iacona, A. 2010. Truth Preservation in Any Context. *American Philosophical Quarterly* 47: 191–199.
- Kamp, H. 1971. Formal Properties of “now”. *Theoria* 37: 227–73.
- Kaplan, D. 1989a. Afterthoughts. In *Themes from Kaplan*, edited by J. P. J. Almog and H. Wettstein, 565–614. Oxford University Press.
- Kaplan, D. 1989b. Demonstratives. In *Themes from Kaplan*, edited by J. P. J. Almog and H. Wettstein, 481–563. Oxford University Press.
- King, J. C. 2003. Tense, Modality and Semantic Value. *Philosophical Perspectives* 17 (1): 195–245.
- Lepore, E., and K. Ludwig. 2002. What is Logical Form? In *Logical Form and Language*, 54–90. Oxford University Press.

<sup>18</sup> Iacona (Forthcoming) outlines an account of ‘more than half of’, ‘most’, ‘many’ and ‘few’ along these lines and shows how the first-order expressibility of these expressions can be established.

- Peters, S., and D. Westerståhl. 2006. *Quantifier in language and logic*. Oxford University Press.
- Prior, A. N. 1957. *Time and modality*. Oxford University Press.
- Recanati, F. 2007. *Perspectival thought*. Oxford University Press.
- Russell, B. 1998. *The philosophy of logical atomism*. Open Court.
- Stanley, J. 2000. Context and Logical Form. *Linguistics and Philosophy* 23: 391–434.
- Wittgenstein, L. 1992. *Tractatus logico-philosophicus*. Routledge.
- Yagisawa, T. 1993. Logic Purified. *Noûs* 27 (4): 470–486.

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